Energy barrier to decoherence

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We propose a ground-state approach to realizing quantum computers. This scheme is time-independent and inherently defends against decoherence by possessing an energy barrier to excitation. We prove that our time-independent qubits can perform the same algorithms as their time-dependent counterparts. Advantages and disadvantages of the time-independent approach are described. A model involving quantum dots is provided for illustration.

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To realize the theoretical potential of quantum computation [1–3], it is essential to confront the difficult task of designing and constructing a functioning quantum computer [4]. An impressive body of quantum error correction literature has shown how, given a limitless supply of qubits and gates with fixed (small) decoherence, the qubits can be connected to execute quantum computation algorithms of arbitrary complexity [5–7]. The daunting problem of supplying the qubits and gates, however, remains unsolved. After an intensive effort to find physical implementations [8–21], it remains unclear whether decoherence can be reduced enough to make a useful quantum computer. In this paper, instead of a specific implementation, we suggest an approach to the problem that inherently defends against decoherence with an energy barrier [22]. This is achieved by proposing that a qubit be constructed not as a two-state quantum system developing through \(N\) unitary time evolutions but, instead, as a time-independent quantum system developing through a \(2(N + 1)\)-dimensional Hilbert space.

To formulate this time-independent approach to quantum computation, let us first review the usual time-dependent approach. Suppose that a quantum algorithm requires the wave function of the electron to develop through a Hamiltonian in the \(2^{(N+1)}\)-dimensional Hilbert space.

The complete Hamiltonian for a calculation with a specific \(N\) can be written as \(H = \sum_{i=1}^{N} h^i(U_i)\), where

\[
h^i(U) = \epsilon [ C^i_{i-1} C_{i-1} + C^i_{i} C_{i} -(C^i_{i} U C_{i-1} + \text{H.c.)} ]
\]

and the constant energy \(\epsilon\) defines the energy scale of the Hamiltonian. This Hamiltonian is positive semidefinite and has two degenerate ground states of zero energy. The two ground states both satisfy Eq. (2), but one has \(P_0 |\Psi\rangle = |\text{vac}\rangle\) and the other has \(P_0 |\Psi\rangle = |\text{vac}\rangle\). The two possibilities correspond to different possible input values. The complete Hamiltonian for a calculation with a specific

\[
|0\rangle
\]

\[
|1\rangle
\]

FIG. 1. Electron shared between two quantum dots constitutes a hypothetical qubit.
required to influence any other aspect of the array since the 
U_i singles nondegenerate ground state.

2 tunneling matrix elements that connect the states in row 
i. The four values in the matrix U_i would determine the four 
tunneling matrix elements that connect the states in row i − 1 to the states in row i. The four values of U_i would not be 
required to influence any other aspect of the array since the 
operator (3) has an appealing modular character: the unitary 
matrix U_i only enters the Hamiltonian through matrix ele-
ments between states on rows i−1 and i. The small pertur-
bage added to break the degeneracy of (3) and select input 
would be supplied physically by applying a voltage to one of 
the dots in row 0. After application of the perturbation, the 
system would be annealed to its ground state. The output of 
the calculation would be obtained by measuring on which of 
the dots at row N the electron can be found.

What are the advantages of this ground-state approach to 
quantum computation? Most importantly, it possesses a cer-
tain robustness against decoherence. Certainly, time-
dependent or time-independent perturbations of the Hamil-
thonian could introduce errors into the calculation. Static 
perturbations due to imperfect implementation of the requi-
site Hamiltonian will adversely influence the ground state. If 
a ground-state quantum computer is to function, such time-
dependent sources of decoherence must first be removed 
by testing and refining the computer apparatus. Thus 
ground-state quantum computation does not require time-
dependent control of a system, but it does demand fine tun-
ability of a static Hamiltonian. The required precision of 
implementation is as high as it is in the case of time-
dependent quantum computation.) However, the inevitable, 
uncontrollable time-dependent perturbations from the envi-
ronment only influence the calculation if they excite the sys-
tem out of the ground state. In a traditional quantum compu-
ter only changes when switching from one algorithm to another. 
(For the quantum dot example that we give, this alteration 
could be implemented by using intermediate dots of adjust-
able voltage to control interdot tunneling rates.) This could 
ease somewhat the problem of realizing a quantum com-
puter.

There are two disadvantages to our ground-state approach 
that should be pointed out. First, rather than having a small 
system that evolves in time through N time steps before de-
coherence sets in, we require N + 1 copies of the small sys-
tem. This could be an inefficient use of hardware. (Although 
it must be noted that a small time-dependent system can 
require a tremendous amount of hardware to control its evolu-
tion.) Second, to detect the results of a calculation, an elec-
tron must be measured in row N. Since this does not occur 
with certainty, there is a chance that the system will have to be 
annealed to the ground state again after an unsuccessful 
measurement. However, it is possible to reduce this problem 
drastically by adjusting \( \hbar^N(U) \) in the Hamiltonian (3) to read

\[
\hbar^N(U) = \epsilon \left[ C_{N-1}^l C_{N-1}^l + \frac{1}{\lambda^2} C_N^l C_N^l - \frac{1}{\lambda} (C_N^l U C_{N-1}^l + \text{H.c.}) \right].
\]

This will change the ground state so that, while it still satis-
fies Eq. (2), the probability that the electron is at the last row 
is enhanced by a factor of \( \lambda^2/(\lambda^2 + (N-1)^2) \). For \( \lambda \sim N \), the 
probability of an unsuccessful measurement becomes small.

Up to this point, we have focused upon a single qubit. To 
perform useful quantum computations, we must consider the 
M qubit case. It is natural to redefine \( |\Psi\rangle \) to be an M qubit 
state, \( \mathcal{U}_f \) to be a \( 2^M \) by \( 2^M \) unitary operator specified by the 
algorithm, \( P_i = \prod_a C_{a,i}^l C_{a,i} \) to be a multiple qubit projector, 
and \( A_{i,i-1} = \prod_c C_{a,i} C_{a,i-1} \) to be a multiple qubit mover, 
where the index \( a \) specifies the qubit upon which a given 
operator acts. Unfortunately, if we simply insert these redefi-
nitions into Eq. (2), the result constrains but does not fully 
specify the M qubit wave function. It contains no informa-

FIG. 2. Quantum dot qubit develops in time in accordance with 
the algorithm.

input value does not possess this degeneracy, but rather in-
cludes a small perturbation term so that the energies of 
\( c_{0,0}^\dagger |\text{vac}\rangle \) and \( c_{0,1}^\dagger |\text{vac}\rangle \) differ slightly. As a result, there is a 
single nondegenerate ground state.

Physically, the Hamiltonian (3) would be realized in the 
quantum dot system by fabricating the dots such that they 
possess appropriate on-site and tunneling matrix elements. 
The four values in the matrix \( U_i \) would determine the four 
tunneling matrix elements that connect the states in row i − 1 to the states in row i. The four values of \( U_i \) would not be 
required to influence any other aspect of the array since the 
operator (3) has an appealing modular character: the unitary 
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nitions into Eq. (2), the result constrains but does not fully 
specify the M qubit wave function. It contains no informa-

FIG. 3. Electron in an array of quantum dots forms a single 
qubit. Amplitude of wave function develops through array in accordance with the algorithm. Lines indicate tunneling paths.
tion about those terms in the many-body wave function that are annihilated by every projector \( P_i \), i.e., terms in which not all electrons are at the same row of the computer. We must posit a development equation that will specify all these terms (which have no analog in time-dependent quantum computation), but will result in a \( |\Psi\rangle \) that is the ground state of a simple Hamiltonian. This is achieved by defining \( |\Psi\rangle \) to be the ground state of a hypothetical computer, with only \( j \) of the actual computer’s \( N \) rows, and requiring

\[
|\Psi\rangle = \prod_{a=1}^{M} (1 + C_{a,j} U_{a,j} C_{a,j-1}) |\Psi^{j-1}\rangle. \tag{5}
\]

It is straightforward to check that this more specific equation implies the multiple qubit redefinition of Eq. (2) and the appropriate Hamiltonian is just the sum of the familiar one qubit Hamiltonians (3).

With Eq. (5), we are now in a position to include the essential two qubit controlled-NOT gate. Assume the algorithm specifies as the \( j \)th operation \( U_j \) a controlled-NOT of qubit \( B \) by qubit \( A \) and unitary operations \( U_{a,j} \) on the other qubits \( a \neq A,B \). The desired multiple qubit redefinition of Eq. (2) will still hold if we modify Eq. (5) at row \( j \) to read

\[
|\Psi\rangle = [1 + c_{A,j \bar{0}} c_{A,j - 1 \bar{0}} (1 + C_{B,j} C_{B,j-1}) \\
+ c_{A,j \bar{1}} c_{A,j - 1 \bar{1}} (1 + C_{B,j} N C_{B,j-1})] \\
\times \prod_{a \neq A,B} (1 + C_{a,j} U_{a,j} C_{a,j-1}) |\Psi^{j-1}\rangle, \tag{6}
\]

where \( N \) is the two-by-two NOT matrix (the Pauli matrix \( \sigma_z \)). The actual ground state will satisfy this requirement if the Hamiltonian is a sum of one-qubit Hamiltonians (3) and a two-body term of the form

\[
h_{A,B}(CNOT) = e C_{A,j-1}^\dagger C_{A,j-1} C_{B,j} C_{B,j-1} + h_{A}(I) C_{B,j-1}^\dagger C_{B,j-1} + C_{A,j \bar{0}}^\dagger C_{A,j \bar{0}} h_{B}^\dagger(I) + c_{A,j \bar{1}} c_{A,j \bar{1}} h_{B}^\dagger(N). \tag{7}
\]

Physically, in the quantum dot realization, the terms in Eq. (7) correspond to having the location of one qubit influence (by the Coulomb interaction) the tunneling matrix elements of another qubit.

We note in conclusion that the idea of using a time-independent Hamiltonian for quantum computation has been raised before [24, 25]. This prior work, however, aimed to perform the usual time-dependent quantum computation using a static “cursor Hamiltonian.” Such an approach (i) requires tailoring a Hamiltonian with specific three-particle interactions and (ii) demands time-dependent state preparation and measurement. In addition, it (iii) is particularly susceptible to decoherence in the form of unintended reflections [26] that are not relevant to our design. The “‘cursor Hamiltonian” is probably, therefore, unfeasible. Our approach does not suffer from these basic problems, but its ultimate viability can only be assessed by developing specific implementations. Our quantum dot array implementation seems unrealistic (although it is encouraging to note that a classical computation scheme using coupled quantum dots has been implemented [27, 28]). However, many other possibilities could be envisioned, e.g., the \( 2(N+1) \) states of a single qubit could take different locations in momentum space rather than different locations in real space. Perhaps it will even turn out to be fruitful to combine the approach we describe here with other ways of handling decoherence [5–7,29].

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